

# BUBBLE GROWTH AS A DETONATION

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## Abstract

In the case of spherically symmetric chemical burning, only very special hydrodynamical detonation solutions exist. These are the so called Jouguet detonations, in which the burning front moves at sound velocity with respect to the burnt matter. Usually it is believed that the situation is similar in the case of bubble growth in cosmological phase transitions. In this paper it is shown that actually a much larger class of detonation solutions exists in cosmological phase transitions.

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# 1 Introduction

First order phase transitions in cosmology — probably at least the electroweak phase transition and the quark-hadron phase transition — proceed through the supercooling of the high-temperature phase and the nucleation of supercritical bubbles of the low-temperature phase. The nucleated bubbles start to grow. Assuming that this process is mainly hydrodynamical, the bubbles can grow either as deflagrations or as detonations. In a large part of parameter space, the bubbles grow as deflagrations which must be preceded by shock waves to satisfy the boundary conditions. However, if the supercooling is considerable the bubbles can grow as detonations [3, 4].

Both deflagration and detonation solutions can be further subdivided into three categories which have different qualitative features [1, 2]. The categories are called the strong, the weak and the Jouguet detonations and deflagrations. These different types of processes are characterized by the velocity of the combustion front with respect to the matter behind the front, the Jouguet processes being those for which matter flows out of the interface at sound velocity. In any attempt to account for the precise mechanism of a first order phase transition one should in the very beginning be aware of the category to which the growing bubbles belong. However, to be able to do so one must give up the idealization of viewing the phase transition surface as a discontinuity and to study its microscopic physics. In the context of chemical burning this problem is well understood [1, 2]. In the case of cosmological phase transitions, less is known.

In this paper we show what the relevant categories of detonations are, and we also review the situation in the case of deflagrations. For chemical burning taking place as a deflagration, it is well known that strong deflagrations are impossible. This means that the burning front cannot move supersonically with respect to the burnt matter. This result will be restated in section 4 in the context of cosmological phase transitions. However, nothing more can be said about the velocity of the phase transition surface without microscopic calculations such as those performed in refs. [5]–[7] for the electroweak phase transition and in ref. [8] for the quark-hadron phase transition. For bubbles growing as detonations the situation is different and quite interesting. Namely, in the case of chemical burning the bubbles can only grow as Jouguet detonations [1, 2]. This remarkable result (called the Chapman-Jouguet hypothesis) means in particular that the velocity of the combustion front is completely determined by energy-momentum conservation and by the boundary conditions, and no degrees of freedom are left to the microscopic physics of the burning surface. In the case of phase transitions, there has been some confusion about the validity of this result. Steinhardt has argued in ref. [9], using the same arguments as Landau and Lifshitz used for chemical burning in ref. [1], that the Chapman-Jouguet hypothesis should be valid. His proof was originally in 1+3 dimensions but it is very easily “extended” to 1+1 dimensions as well. The authors in for instance refs. [3, 6] were also under the impression that only the Jouguet

detonations are possible, though at least in refs. [4, 10] the weak detonations in 1+1 dimensions are accepted as well. There seems to have prevailed the belief that the dimension of the space would have something to do with the validity of Steinhardt's results. In refs. [11, 12] the authors use Steinhardt's results in the 1+3-dimensional context, and also in ref. [13] there is some uncertainty about the possibility of the different kinds of detonations. It is the purpose of this paper to show that due to qualitative differences between chemical burning and phase transitions, the Chapman-Jouguet hypothesis does *not* hold in the case of phase transitions and therefore weak detonations are possible, in addition to the Jouguet detonations.

The plan of the paper is as follows. In section 2 the relativistic hydrodynamics of first order phase transitions is reviewed. In section 3, it is proved that strong detonations are not possible as a means of bubble growth in phase transitions. In section 4, it is shown that strong deflagrations are impossible as well. In section 5 we address the question of weak detonations and show that these are a natural means of bubble growth in phase transitions. The conclusions are in section 6.

## 2 The hydrodynamics of bubble growth

Consider a spherically expanding combustion front. Locally the combustion front looks planar. If the front velocity is constant, we can Lorentz transform into the rest frame of the front. In this frame, we denote the matter ahead of the front (the unburnt or the "quark" matter) with subscript 1 and the matter behind the front (the burnt or the "hadron" matter) with subscript 2. It is a very good first idealization to treat the front as a discontinuity. The strong deviation from thermodynamical equilibrium attached with the burning of the quark phase to the hadron phase and all the entropy production is confined to this discontinuity, and outside it the energy-momentum tensor is that of an ideal fluid:  $T^{\mu\nu} = w u^\mu u^\nu - p g^{\mu\nu}$ . Here  $w = e + p$  is the enthalpy density,  $e$  is the energy density,  $p$  is the pressure and  $u^\mu = (\gamma, \gamma\mathbf{v})$  is the four-velocity of the fluid. The energy-momentum conservation across the discontinuity yields the equations

$$\begin{aligned} w_1 \gamma_1^2 v_1 &= w_2 \gamma_2^2 v_2 \\ w_1 \gamma_1^2 v_1^2 + p_1 &= w_2 \gamma_2^2 v_2^2 + p_2 . \end{aligned} \tag{1}$$

The non-negativity of entropy production is expressed as

$$s_2 \gamma_2 v_2 \geq s_1 \gamma_1 v_1 , \tag{2}$$

where  $s$  is the entropy density. For generality, we also assume that there is a conserved quantity (its density is denoted by  $n$ ), which is in thermodynamical equilibrium with the rest of the fluid. Then we have the additional equation

$$n_1 \gamma_1 v_1 = n_2 \gamma_2 v_2 \equiv j . \tag{3}$$

In the quark-hadron phase transition the conserved quantity is the baryon number. In the electroweak phase transition no such conserved quantity exists, but one can always add a fictitious conserved quantity (a “tracer”) to the fluid as long as it does not show in the equation of state. To see how this can be done, consider a problem in which there is no conserved quantity,  $\mu = 0$ . Then the equation of state is  $p = p(T)$  and the complete solution of the hydrodynamical problem (possibly involving discontinuities) consists of the functions  $T(t, \mathbf{x})$ ,  $\mathbf{v}(t, \mathbf{x})$  and  $\phi(t, \mathbf{x})$ . Here  $\phi(t, \mathbf{x})$  is the order parameter and it is needed when the problem contains a phase transition. At the initial time  $t = 0$ , choose an arbitrary function  $n(0, \mathbf{x})$ , for instance  $n(0, \mathbf{x}) = 1$  in some units. Then one can integrate the first order partial differential equation  $\partial_\mu(nu^\mu) = 0$ , where  $u^\mu$  is now a known function of  $t$  and  $\mathbf{x}$ . The solution  $n(t, \mathbf{x})$  is the required conserved quantity. Although  $\mu = 0$ , the quantity  $n$  enters our thermodynamical equations, because it is natural in the present context to formulate the thermodynamical identities using the entropy per tracer  $\sigma = s/n$ . Then the quantity  $x = w/n^2$  will prove to be very useful, see below. Notice also that the arbitrariness of the initial condition for  $n(t, \mathbf{x})$  means just that it is equivalent to say that the entropy per one tracer is increased, or that the entropy per ten tracers is increased. Finally, in a thermal fluid with vanishing chemical potential, the true physical particle density  $\tilde{n}$  of all the massless relativistic particles (and antiparticles) is proportional to entropy and therefore satisfies the inequality  $\partial_\mu(\tilde{n}u^\mu) \geq 0$  in contrast to the equation satisfied by the tracer. The inequality sign is obeyed at least at the discontinuities. In the following, when we speak of baryons we mean just some conserved quantity, possibly the tracer.

If one does not want to use the tracer in the case  $\mu = 0$ , it is still possible to repeat the following analysis almost as such, see ref. [14]. The quantity  $x$  only has to be defined in another way.

To describe the state of the fluid on both sides of the discontinuity, two intensive thermodynamical variables are needed. It is conventional to take as these the pressure  $p$  and the variable  $x = w/n^2$  [1]. Then the state of the fluid is represented by a point in the  $(x, p)$  plane, and the above continuity conditions relate the two points. Explicitly, from equations (1) and (3) we get the equations

$$-j^2 = (p_2 - p_1)/(x_2 - x_1) \quad (4)$$

$$w_2x_2 - w_1x_1 = (p_2 - p_1)(x_2 + x_1). \quad (5)$$

Given the point  $(x_1, p_1)$  and the equations of state of both phases, the enthalpy  $w$  can be expressed in terms of  $x$  and  $p$  and from equation (5) the curve  $p_2 = p_2(x_2)$  can be solved. This curve is called the detonation adiabat. If both phases are ascribed the same equation of state, then eqs. (4) and (5) do not describe a combustion front but a shock front within one phase. In this case the curve  $p_2 = p_2(x_2)$  is called the shock adiabat or the Taub adiabat.

To illustrate the general structure of the detonation and the shock adiabats in

the  $(x_2, p_2)$ -plane, we need to know something about the equations of state of the burnt and the unburnt matter. In view of the comments after eq. (3), we could use for instance the bag equation of state<sup>1</sup> in this purpose as was done in ref. [9], although this would mean that the chemical potential vanishes and hence there is no true conserved physical quantity. However, we can easily work a bit more generally to understand some of the underlying assumptions. Consider first the shock adiabat of the unburnt matter through the point  $(x_1, p_1)$ . Taking either the equation of state in ref. [15, eq. (2.3)] with non-zero chemical potential, or the bag equation of state, we obtain the relation  $w_1 = 4p_1 + 4B$ . Then from eq. (5) the equation of the shock adiabat becomes

$$[p_s + \frac{1}{3}(p_1 + 4B)][x_s - \frac{1}{3}x_1] = \frac{8}{9}x_1(p_1 + B) . \quad (6)$$

The curve  $p_s = p_s(x_s)$  is a hyperbola and it is drawn schematically in figure 1 with dashed line. Consider then the detonation adiabat. It is essential that the detonation and the shock adiabats never cross and that the detonation adiabat always lies above the shock adiabat. To prove the first claim, notice that it follows from equation (5) that if the two adiabats crossed, then the final states would have exactly the same enthalpy density, particle density and pressure. This is not possible because the two phases have a different equation of state:  $w_1(n_2, p_2) \neq w_2(n_2, p_2)$ . For instance, at the critical temperature in the quark-hadron phase transition we have  $s_1(T_c, \mu_c) > s_2(T_c, \mu_c)$  because of the latent heat and  $n_1 > n_2$  as is seen in ref. [15, fig. 3], so that  $w_1 = T_c s_1 + \mu_c n_1 > w_2$ . Specifically, for the bag equation of state we have  $w_1(p_2) - w_2(p_2) = 4B$ . To prove the second claim, consider equation (4) in the limit that  $x_2$  approaches  $x_1$  from below. It is seen that  $p_2 > p_1$  for all  $x_2 < x_1$ . Therefore, because the adiabats cannot cross even at  $x_2 = x_1$ , the point  $(x_1, p_2)$  must lie above  $(x_1, p_1)$  and the claim is proved. With this knowledge, the general structure of the detonation adiabat can be drawn. In many cases (e.g. in the bag equation of state in the limit  $B \rightarrow 0$ ) it is even so that tuning some parameters makes the equations of state of the burnt and the unburnt phases the same so that near this limit the detonation adiabat resembles greatly the shock adiabat. To prove more precise statements about the detonation adiabat, one should combine the non-relativistic analysis in refs. [1, 2] with the relativistic analysis in ref. [16]. The detonation adiabat is represented schematically in figure 1 with solid line.

We can now enumerate the different qualitative processes by which the bubble growth can proceed. From equation (4) it is known that the line in the  $(x, p)$ -plane through points  $(x_1, p_1)$  and  $(x_2, p_2)$  always has a negative slope. Therefore, there are two kinds of solutions: either  $p_2 > p_1$  or  $p_1 > p_2$ . Solutions of the first kind are detonations; examples of these are the points B and C in figure 1 where the dash-dotted

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<sup>1</sup>In the bag equation of state, the pressures and the energy densities are  $p_1 = a_1 T_1^4 - B$ ,  $e_1 = 3a_1 T_1^4 + B$ ,  $p_2 = a_2 T_2^4$  and  $e_2 = 3a_2 T_2^4$ . Here  $a_1 = a_2 + B/T_c^4$  and  $B$  is the bag constant.

line intersects the detonation adiabat. Solutions of the second kind (points F and G in figure 1) are deflagrations. There are three kinds of both detonations and deflagrations. Point C is a *strong* detonation, point B is a *weak* detonation and point E is a *Jouguet* (or Chapman-Jouguet) detonation. Similarly, point G is a strong deflagration, point F a weak deflagration and point H a Jouguet deflagration. From energy-momentum conservation alone none of these processes can be excluded. Notice, however, that the entropy condition (2) does in many cases rule out the whole family of detonations [4, fig. 17] but it does not categorically rule out any of the different types of detonations: when the phase transition is preceded by sufficient supercooling, all the processes are in principle possible.

The different types of processes can be characterized by the velocities at which matter flows into and out of the discontinuity. To be able to do so, note first that the slope of a shock adiabat through any point in the  $(x, p)$ -plane is the quantity  $-n^2\gamma_s^2v_s^2$ , where  $v_s$  is the local sound velocity [16]. Second, consider the detonation adiabat through the point  $(x_2, p_2)$ , which is hereafter called point 2. This curve is *not* the same as the shock adiabat through point 2. It can be seen using eqs. (4) and (5) and the relativistic versions in ref. [16] of the results of ref. [1, §129] that the detonation and the shock adiabats through point 2 cross at exactly two points, namely the points B and C in figure 1 when point 2 is either of these, and that the slopes of the detonation and the shock adiabat agree only if point 2 is point E. Therefore, contrary to Steinhardt's claim in ref. [9], the slope of the detonation adiabat does not give the local value of the quantity  $-n^2\gamma_s^2v_s^2$ . However, when the slope of the detonation adiabat is bigger than the slope of the straight line between the points  $(x_1, p_1)$  and  $(x_2, p_2)$ , then also the slope of the shock adiabat at point 2 is bigger than the slope of this line, and vice versa. Because the slope of the straight line between  $(x_1, p_1)$  and  $(x_2, p_2)$  is by eq. (4) just  $-j^2 = -n_1^2\gamma_1^2v_1^2 = -n_2^2\gamma_2^2v_2^2$ , we see directly from figure 1 the following characteristics of the different processes:

- strong detonation:  $v_1 > v_{s1}$ ,  $v_2 < v_{s2}$
- Jouguet detonation:  $v_1 > v_{s1}$ ,  $v_2 = v_{s2}$
- weak detonation:  $v_1 > v_{s1}$ ,  $v_2 > v_{s2}$
- strong deflagration:  $v_1 < v_{s1}$ ,  $v_2 > v_{s2}$
- Jouguet deflagration:  $v_1 < v_{s1}$ ,  $v_2 = v_{s2}$
- weak deflagration:  $v_1 < v_{s1}$ ,  $v_2 < v_{s2}$

Here  $v_{s1}$  is the local sound speed of the unburnt matter at the thermodynamical state  $(x_1, p_1)$  and  $v_{s2}$  is that of the burnt matter at  $(x_2, p_2)$ . For instance, a weak detonation is supersonic relative to the matter both behind and in front of the surface. To relate

the velocities  $v_1$  and  $v_2$  to the propagation velocity of the combustion front, boundary conditions must be considered. The boundary conditions of an expanding bubble are that the matter far ahead of the phase transition surface (where no information of the phase transition has yet arrived) and far behind the phase transition surface (inside the bubble of the low temperature phase) is at rest. To satisfy these boundary conditions, the flow profile of a growing bubble consists of several regions [17]. In the case of a detonation, the matter ahead of the combustion front is at rest (see e.g. fig. 2). Therefore the velocity of the detonation front  $v_{\text{det}}$  is the velocity  $v_1$  at which matter flows *into* the combustion front. In the case of a deflagration, the matter behind the combustion front is at rest (see e.g. fig. 3). Therefore the velocity of the deflagration front  $v_{\text{def}}$  is the velocity  $v_2$  at which matter flows *out of* the combustion front. Hence, for instance, weak deflagrations move subsonically.

It will be useful to notice that formally, treating every front as a discontinuity, a detonation front is equivalent to a shock front which is followed by a deflagration front moving with the same velocity as the shock front. We prove this for planar fronts. Consider a process in which the state of the fluid changes from  $(x_1, p_1)$  to the intermediate state  $(x_0, p_0)$  and then from  $(x_0, p_0)$  to  $(x_2, p_2)$ . From equation (5) we get

$$\begin{aligned} w_0 x_0 - w_1 x_1 &= (p_0 - p_1)(x_0 + x_1) \\ w_2 x_2 - w_0 x_0 &= (p_2 - p_0)(x_2 + x_0) \quad . \end{aligned}$$

Summing these together we see that the points  $(x_1, p_1)$  and  $(x_2, p_2)$  satisfy eq. (5) if

$$\frac{p_0 - p_1}{x_0 - x_1} = \frac{p_2 - p_1}{x_2 - x_1} .$$

Therefore, the point  $(x_0, p_0)$  is just the point D in figure 1 and the route AC is equivalent to the route ADC. The part AD is a shock and the part DC a deflagration. From the above it follows that the baryon flux  $j$  attains the same value at the shock front and at the deflagration front. But then matter flows out of the shock front at the same velocity as it flows into the deflagration front, which in the planar case implies that the shock and the deflagration fronts move with the same velocity. In a similarity solution, which is valid after the early stages of bubble growth, the fronts are therefore at the same place. Finally, because matter flows out of a weak detonation front with supersonic velocity, a *weak* detonation is equivalent to a shock and a *strong* deflagration and a strong detonation is equivalent to a shock and a weak deflagration.

### 3 The impossibility of strong detonations

Of the presented six mechanisms of a phase transition, only some are actually realized as physical processes. To begin with, an expanding bubble of the low temperature phase

*cannot* grow as a strong detonation. This can be seen even on very general arguments. Basically, in a strong detonation there are not enough degrees of freedom to adjust to arbitrary boundary conditions [2] and specifically not to those of an expanding bubble. We will next prove the impossibility of strong detonations very explicitly, too, since this proof will provide us with valuable information. For simplicity, our proof will be in 1+1 dimensions. An analogous proof in 1+3 dimensions has been given in ref. [1] for the non-relativistic case and with some modifications in ref. [9] for the relativistic case.

The equations governing the evolution of a bubble are the jump conditions in eqs. (4) and (5), the energy-momentum conservation  $\partial_\mu T^{\mu\nu} = 0$  and the charge conservation  $\partial_\mu(nu^\mu) = 0$ . Denote by  $\sigma$  the entropy per baryon. Projecting the equation  $\partial_\mu T^{\mu\nu} = 0$  in the direction of and in the direction perpendicular to the flow velocity  $u^\mu$ , the energy-momentum and charge conservation equations can be written in the form

$$\begin{aligned}\partial_t(n\gamma) + \partial_x(n\gamma v) &= 0 \\ \partial_t\sigma + v\partial_x\sigma &= 0 \\ \partial_tv + v\partial_xv &= -(\partial_xp + v\partial_tp)/w\gamma^2 \quad .\end{aligned}$$

For a similarity solution depending only on the variable  $\xi = x/t$  these equations reduce to

$$(\xi - v)n' = (1 - \xi v)n\gamma^2v' \quad (7)$$

$$(\xi - v)\sigma' = 0 \quad (8)$$

$$(\xi - v)v' = (1 - \xi v)p'/w\gamma^2 \quad (9)$$

where the prime denotes differentiation by  $\xi$ . From eq. (7) it follows that the solution cannot be  $v = \xi$ . Then eq. (8) implies that  $\sigma' = 0$ . But now using simple thermodynamics we can write  $p'$  in terms of  $n'$ :

$$\frac{dp}{d\xi} = \left( \frac{\partial p}{\partial e} \right)_\sigma \left( \frac{\partial e}{\partial n} \right)_\sigma \frac{dn}{d\xi} = v_s^2(w/n) \frac{dn}{d\xi} .$$

Then eqs. (7) and (9) give an equation for  $v(\xi)$  alone, which is trivially solved to yield two solutions, namely  $v' = 0$  and

$$v_s(\xi) = \frac{\xi - v(\xi)}{1 - \xi v(\xi)} . \quad (10)$$

Putting these two types of solutions together, we get a picture of the bubble growing as a detonation. For  $0 \leq \xi \leq v_s$ , the fluid is at rest. At  $\xi = v_s$  starts the rarefaction solution, eq. (10). Then there are two possibilities: either the phase transition discontinuity comes straight after the rarefaction solution, in which case it follows from eq. (10)

that matter flows out of the discontinuity at exactly the sound speed, or there is an area of constant velocity between the rarefaction and the discontinuity. In the latter case matter flows out of the discontinuity at a velocity greater than  $v_s$  because the right hand side of eq. (10) is a growing function of  $\xi$  for constant  $v$ . The former possibility is a Jouguet detonation, the latter a weak detonation. These are shown schematically in figure 2, together with the corresponding 1+3 -dimensional detonation bubbles. Hence, strong detonations are not possible.

## 4 The impossibility of strong deflagrations

Another type of solutions which are not realized in nature is strong deflagrations. Performing some causal analysis near the interface, one can conclude that strong deflagrations would not be mechanically stable and hence they cannot exist [1, §131]. Another way to exclude these is to give up the idealization of viewing the phase transition surface as a discontinuity and to study its microscopic structure. The essential feature which makes strong deflagrations impossible is that thermodynamical variables — to the extent that they can be used in this region — change monotonically and continuously between the two phases [2]. Then in a strong deflagration the state of matter would slide from point A in figure 1 to point G. Assume that inside the burning zone matter can be described as a changing mixture of the low-temperature phase and the high-temperature phase, so that the energy-momentum tensor is that of an ideal fluid<sup>2</sup>. Then equation (5) is satisfied at every intermediate point  $(\tilde{x}_2, \tilde{p}_2)$  between A and G. This means that at point F the state of the system characterized by  $\tilde{x}_2, \tilde{p}_2, \tilde{w}_2$  (from eq. (5)) and  $\tilde{n}_2$  (from the equation  $x = w/n^2$ ) has not only the same variables  $x_2$  and  $p_2$  as the low-temperature phase at this point but also the same  $w_2$  and  $n_2$ , since the detonation adiabat is just the solution of eq. (5). Because the equation of state fixes the relation  $w_2 = w_2(n_2, p_2)$ , the system actually already has to be in the low-temperature phase at point F. But then the transition from point F to point G would be completely equivalent to a rarefaction shock, since it was noted earlier that the shock adiabat through point F crosses the detonation adiabat also at point G. Since entropy increase does not allow rarefaction shocks, there cannot be any microscopic mechanism by which to move from F to G, *i.e.* strong deflagrations are not possible. Notice, however, that entropy increase does allow a similarity rarefaction solution, which is not a discontinuity. Therefore it seems possible that if a strong deflagration could somehow momentarily be forced to exist, it would tend to split into a similarity rarefaction solution and a weak deflagration.

To give an example of a typical deflagration solution, we cite results from ref. [18]. There a simple model is given for first order phase transitions taking place in relativistic

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<sup>2</sup>When there is a microscopic order parameter field, one has to make the proof somewhat differently.

matter. The model includes one phenomenological parameter  $\Gamma$  of dimension  $\text{GeV}^{-1}$  which is a sort of dissipation constant and fixes the entropy production at the phase transition surface. The dissipation constant is roughly proportional to the collision time  $\tau_c$ . Small  $\Gamma$  means large friction and large  $\Gamma$  small friction at the interface. In this model, it turns out that in deflagration solutions the temperature and other thermodynamical variables really behave monotonically at the phase transition surface, see fig. 3. Accordingly, only weak deflagrations (and as a limiting case of these, Jouguet deflagrations) have been found.

## 5 Weak detonations

We now turn to weak detonations. It has been argued in refs. [1, 2] for the non-relativistic case of *chemical burning* and in ref. [9] for the case of phase transitions in relativistic matter that weak detonations are impossible. Then the Jouguet detonation would be left as the only mechanism for bubble growth as a detonation. This is called the Chapman-Jouguet hypothesis. To see how this arises, consider chemical burning. The rate of chemical burning increases rapidly with increasing temperature, often as  $\exp(-U/T)$  where  $U$  is some constant factor. Therefore, heat must be supplied to the unburnt matter before the burning can begin. In deflagrations, this happens by thermal conduction from the burning zone where heat is liberated on account of the exothermic character of the reaction. This mechanism is, however, so slow that the velocity of the burning front is subsonic [1]. To make a detonation, an entirely different means of raising the temperature is needed, and this can be achieved by a strong shock wave which ignites the burning. The shock front compresses the matter so that the temperature and the pressure rise dramatically, and then the burning zone follows in which pressure again decreases (the temperature increases even in the burning zone, which is possible because the chemical potential is non-vanishing). This means that the formal equivalence of a detonation to a shock and a deflagration, which was proved above, would also describe the true microscopic structure of the detonation front. In figure 1, this means that matter first jumps from point A to point D and then slides from D to C as thermodynamical quantities change continuously in the burning zone. Since a shock in which pressure decreases is forbidden by the condition of entropy increase, matter remains in C and cannot continue to B. Another way to put this is that a weak detonation is equivalent to a shock and a strong deflagration, and as strong deflagrations are impossible, so are weak detonations. From these arguments, the would-be mechanism of a detonation in chemical burning must be a strong detonation, point C. But because even strong detonations were above proved to be impossible, the detonation must correspond to point E in figure 1. And this is just the Jouguet detonation, in accordance with the Chapman-Jouguet hypothesis.

The question is now what the microscopic structure of a detonation front in

the case of a *phase transition* actually is. First of all, no preheating such as that caused by the shock front in the case of chemical burning is needed to start the phase transition. Namely, in the case of phase transitions, there is no local “burning rate” which would have to be high enough for the transition to start. The only relevant rate is the rate of nucleation. This is of the form  $\exp(-U/T)$  where  $U$  is not constant but includes singular temperature dependence:  $U \propto 1/(1 - T/T_c)^2$  (in the limit of small supercooling [4, 19]). The lower the temperature is, the faster is the rate of nucleation. Once the nucleation has happened, the growth of the bubble is determined by the equation of motion of the order parameter and by hydrodynamics. It seems highly unlikely that the solution of these equations would include a sharp narrow peak in pressure, because due to the vanishingly small chemical potential in the case of cosmological phase transitions, the peak in pressure would also cause a sharp narrow peak in temperature. Such a peak in temperature is by itself rather unnatural, and it would also make the two minima of the effective potential more degenerate and the potential barrier between the minima higher. Both factors tend to make the tunneling between the minima more difficult. Notice also that in the case of deflagrations, a rise in temperature after the shock front is necessitated by boundary conditions and hydrodynamics, but the supposed temperature peak at the detonation front serves no such purpose. For these reasons, it seems much more natural that the order parameter — if one exists — and the thermodynamical variables change monotonically to their new values in the transition front and the latent heat released in this process fuels the rapid expansion of the detonation front. This means that in figure 1 the state of matter changes directly from point A to point B. As an example, we quote results from the above-mentioned model of ref. [18]. In figure 4, a bubble growing as a detonation is shown. At the detonation front, the temperature really changes monotonically and the solution, accordingly, is a weak detonation. All in all, weak detonations seem to be a natural mechanism for bubble growth in phase transitions.

To be concrete, let us investigate the quark-hadron phase transition in the early universe. The physical parameters describing the first order phase transition are the critical temperature  $T_c$ , the latent heat  $L$ , the surface tension  $\sigma$  and the correlation length  $l_c$ . We take the values  $\sigma = 0.1$  and  $l_c = 6$  in appropriate powers of  $T_c$ . For the latent heat we assume a small value  $L = 0.1T_c^4$ , which results in considerable supercooling so that the nucleation temperature is  $T_f = 0.891T_c$  [8]. With this much supercooling, detonations are possible. With the model of ref. [18] and the phenomenological parameter  $\Gamma$  describing the physics of the phase transition surface, figure 5 results. For small  $\Gamma$  the bubbles grow subsonically as weak deflagrations; for large  $\Gamma$  they grow supersonically as weak detonations. Let us mention in passing that this set of parameters yields a very interesting scenario for the quark-hadron phase transition. The average distance between the nucleated bubbles would be very large, the bubbles would grow as detonations and the phase transition would leave behind it large-scale inhomogeneities

[18].

## 6 Conclusions

When entropy increase allows it, a bubble of the low-temperature phase can grow either as a weak deflagration or a weak detonation (the Jouguet processes are here considered to be limiting cases of these). For weak detonations to be possible, the nucleation must be preceded by considerable supercooling. In weak deflagrations, the velocity of the phase transition surface is smaller than or equal to sound velocity, and in weak detonations, it is larger than sound velocity. In neither case can the energy-momentum conservation and the boundary conditions alone determine the velocity of the phase transition surface: its microscopic physics, for instance the entropy production, must be known before the exact expansion velocity can be calculated. This means that the detonation solutions are qualitatively different in the cases of chemical burning and phase transitions.

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## References

- [1] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, 2nd edition §§85–88, 128–132, 135 (Pergamon Press, Oxford, 1989)
- [2] R. Courant and K. O. Friedrichs, *Supersonic Flow and Shock Waves* (Springer-Verlag, Berlin, 1985)
- [3] M. Gyulassy, K. Kajantie, H. Kurki-Suonio and L. McLerran, Nucl. Phys. **B237**(1984)477
- [4] K. Enqvist, J. Ignatius, K. Kajantie and K. Rummukainen, Phys. Rev. D **45**(1992)3415
- [5] M. Dine, R. G. Leigh, P. Huet, A. Linde and D. Linde, Phys. Rev. D **46**(1992)550
- [6] B. H. Liu, L. McLerran and N. Turok, Phys. Rev. D **46**(1992)2668
- [7] S. Yu. Khlebnikov, Phys. Rev. D **46**(1992)R3223
- [8] K. Kajantie, Phys. Lett. B **285**(1992)331

- [9] P. J. Steinhardt, Phys. Rev. D **25**(1982)2074
- [10] P. Huet, K. Kajantie, R. G. Leigh, B. H. Liu and L. McLerran, SLAC-PUB-5943,  
to be published in Phys. Rev. D
- [11] M. Kamionkowski and K. Freese, Phys. Rev. Lett. **69**(1992)2743
- [12] M. Abney, FERMILAB-PUB-93/098-A
- [13] S. A. Bonometto and O. Pantano, Phys. Rep. **228**(1993)175
- [14] P. Danielewicz and P. V. Ruuskanen, Phys. Rev. D **35**(1987)344
- [15] K. Kajantie and H. Kurki-Suonio, Phys. Rev. D **34**(1986)1719
- [16] K. S. Thorne, Astrophys.J. **179**(1973)897
- [17] H. Kurki-Suonio, Nucl. Phys. **B255**(1985)231
- [18] J. Ignatius, K. Kajantie, H. Kurki-Suonio and M. Laine, Preprint HU-TFT-93-43
- [19] L. D. Landau and E. M. Lifshitz, *Statistical Physics Part 1*, 3rd edition, §162  
(Pergamon Press, Oxford, 1988)

## Figure captions

**Figure 1:** A schematic picture of the detonation and the shock adiabats. Point A is the initial state  $(x_1, p_1)$  of the unburnt fluid. The detonation adiabat shows the possible final states in the  $(x_2, p_2)$ -plane.

**Figure 2:** On the left is a schematic illustration of the velocity profiles in a 1+1 -dimensional weak (solid line) and Jouguet (dashed line) detonation. On the right are the same profiles for the 1+3 -dimensional case [17].

**Figure 3:** A deflagration solution at time  $t = 1800$  after the nucleation in the 1+1 -dimensional model of ref. [18]. The quantities are measured in appropriate powers of  $T_c$  to make them dimensionless. The front on the right is the shock front which has at this time not yet sharpened to an exact discontinuity, and the front on the left is the deflagration front where the phase transition takes place. Both fronts are moving to the right. The velocity of the deflagration front is  $v_{\text{def}} = 0.46$ .

**Figure 4:** A detonation solution at time  $t = 1800$  after the nucleation in the 1+1 -dimensional model of ref. [18]. The quantities are measured in appropriate powers of  $T_c$  to make them dimensionless. The front on the right is the detonation front where the phase transition takes place, and the front on the left is the rarefaction wave. Both fronts are moving to the right. The velocity of the detonation front is  $v_{\text{det}} = 0.72$ . Although the rarefaction wave is very thin in this picture due to the scale of the  $y$ -axis, it cannot be approximated as a discontinuity, see fig. 2.

**Figure 5:** The process by which nucleated bubbles grow in the 1+1 -dimensional model of ref. [18] as a function of the parameter  $\Gamma$  (in units of  $1/T_c$ ) [18]. At approximately  $\Gamma = 10$  the solution changes from a weak deflagration to a weak detonation.